Friday, 8:30 - 9:00

■ FA-01

Friday, 8:30 - 9:00
23.1.5

Opening Session

Plenary session

Friday, 9:00 - 9:45

■ FB-01

Friday, 9:00 - 9:45
23.1.5

Invited Talk 1

Plenary session

Chair: *Marco A. López-Cerdá*, Statistics and Operations Research, Alicante University, Ctra. San Vicente de Raspeig s/n, 3071, Alicante, Spain, marco.antonio@ua.es

1 - Theory and Applications of Degeneracy in Cone Optimization

Henry Wolkowicz, Faculty of Mathematics, University of Waterloo, N2L3G1, Waterloo, Ontario, Canada, hwolkowicz@uwaterloo.ca

The elegant theoretical results for strong duality and strict complementarity for linear programming, LP, lie behind the success of current algorithms. In addition, preprocessing is an essential step for efficiency in both simplex type and interior-point methods. However, the theory and preprocessing techniques can fail for cone programming over nonpolyhedral cones.
We take a fresh look at known and new results for duality, optimality, constraint qualifications, CQ, and strict complementarity, for linear cone optimization problems in finite dimensions. One theme is the notion of minimal representation of the cone and the constraints. This provides a framework for preprocessing cone optimization problems in order to avoid both the theoretical and numerical difficulties that arise due to the (near) loss of the strong CQ, strict feasibility.
Surprisingly, many instances of semidefinite programming, SDP, problems that arise from relaxations of hard combinatorial problems are degenerate (CQ fails). Rather than being a disadvantage, we show that this degeneracy can be exploited. In particular, several huge instances of SDP completion problems can be solved quickly and to extremely high accuracy.

Friday, 10:15 - 12:15

■ FC-03

Friday, 10:15 - 12:15
23.3.4

Optimal Control 1

Contributed session

Chair: *Peter Roebeling*, Department of Environment and Planning, CESAM - University of Aveiro, Campus Universitário de Santiago, Universidade de Aveiro, 3810-193, Aveiro, Portugal, peter.roebeling@ua.pt

1 - Necessary optimality conditions for optimal control problems with discontinuous right hand side

Werner Schmidt, Ernst-Moritz-Arndt-University Greifswald, D-17489, Greifswald, Germany, wschmidt@uni-greifswald.de, *Olga Kostyukova*, *Ekaterina Kostina*

We consider an optimal control problem with discontinuous right-hand side. It is assumed that the system dynamics is switched when system crosses a given surface described by a smooth function dependent on system states. Attention is paid to the situation when optimal trajectory slides on switching surface during nontrivial intervals. Necessary optimality conditions in a form the Maximum Principle are proved. The principle includes new essential conditions. Comparison of the optimality conditions obtained with some other known results is carried out. Illustrative examples are presented.

2 - Invexity in mathematical programming and control problems

Manuel Arana-Jiménez, Estadistica e Invesitigacion Operativa, University of Cadiz, C/Chile, 1, 11002, Jerez de la Frontera, Cadiz, Spain, manuel.arana@uca.es, *Antonio Rufián-Lizana*, *Gabriel Ruiz-Garzón*, *Rafaela Osuna-Gómez*

This communication is focused on the study of optimal solutions for mathematical programming problems and control problems and the properties of the functions, as well as the relationship between these types of optimization problems, from recent published results. KT-invexity has been introduced in control problems, and it is a necessary and suffcient condition in order for a Kuhn-Tucker critical point to be an optimal solution. Recenty, a weaker condition, FJ-invexity, has been proposed, which is characterized by a Fritz John point being an optimal solution for the control problem.

3 - Efficient inter-industry water pollution abatement in linked terrestrial and marine ecosystems

Peter Roebeling, Department of Environment and Planning, CESAM - University of Aveiro, Campus Universitário de Santiago, Universidade de Aveiro, 3810-193, Aveiro, Portugal, peter.roebeling@ua.pt, *Eligius M.T. Hendrix*, *Arjan Ruijs*, *Martijn van Grieken*

Catchment agriculture leads to water pollution, downstream environmental degradation and marine value depreciation. Sustainable development entails balancing of marginal costs and benefits from pollution abatement. As abatement costs differ across agricultural industries and abatement benefits are nonlinear, we explore efficient abatement across industries. Using an optimal control approach with application to cane and cattle industries in Tropical Australia, we show that efficient abatement per industry is dependent on abatement by all industries when marine abatement benefits are non-linear.

■ FC-04

Friday, 10:15 - 12:15
23.3.5

Generalized differentiation and applications

Invited session

Chair: *Vera Roshchina*, CIMA, Universidade de Evora, Colégio Luís Verney, Rua Romão Ramalho, 59, 7000-671, Évora, Portugal, vera.roshchina@gmail.com

1 - Minimizing irregular convex functions: Ulam stability for approximate minima

Michel Théra, Maths-Info, XLIM, UMR-CNRS 6172, 123, Avenue Albert Thomas, 87060, Limoges Cedex, France, michel.thera@unilim.fr

This presentation summarizes a recent joint work with Emil Ernst. Our main objective is to characterize the subclass of those convex lower semicontinuous proper functions bounded below for which the set-valued mapping which assigns to a function in this class, the set of its epsilon-minima is upper semi-continuous. Despite its abstract appearance, this type of stability turns out to be essential in numerical optimization, namely in answering the natural question of defining the largest class of functionals convex lower semicontinuous proper and bounded below for which minimization algorithms exist.

2 - The Directed Subdifferential

Elza Farkhi, School of Math. Sciences, Tel-Aviv University, Haim Levanon Str., 69978, Tel Aviv, elza@post.tau.ac.il, *Robert Baier*

For differences of convex functions the directed subdifferential is introduced as the difference of two embedded convex subdifferentials in the Banach space of directed sets. Basic axioms of subdifferentials and nice calculus rules are established for the directed subdifferential. Its visualization, called Rubinov subdifferential, is a non-empty, generally non-convex set in Rn. Optimality conditions are formulated, minimizers, maximizers and saddle points are distinguished, directions of descent and ascent are identified using the directed and Rubinov subdifferential.

3 - Calculating Known Subdifferentials from the Rubinov Subdifferential

Robert Baier, Department of Mathematics, University of Bayreuth, Chair of Applied Mathematics, D-95440, Bayreuth, Germany, robert.baier@uni-bayreuth.de, *Elza Farkhi*, *Vera Roshchina*

The visualization of the directed subdifferential - defined for differences of convex functions - is the Rubinov subdifferential. This set is usually non-convex and splits into three parts. The relation between these parts and the Dini, Michel-Penot and Clarke subdifferential are discussed. In 2D the Rubinov subdifferential is closely linked to the Mordukhovich one and offers the calculation of various subdifferentials based on simple differences of sets. Several visualizations in examples indicate the connections to other subdifferentials and the advantage of nonconvex subdifferentials.

4 - Subgradient sampling algorithms for nonsmooth nonconvex functions

Adil Bagirov, School of Information Technology & Mathematical Sciences, University of Ballarat, University Drive, Mount Helen, P.O. Box 663, 3353, Ballarat, Victoria, Australia, a.bagirov@ballarat.edu.au

In this talk we will present an algorithm for computation of subgradients of nonsmooth nonconvex functions. Then we demonstrate how this algorithm can be applied to approximate subdifferentials and quasidifferential. We also discuss an algorithm for the computation of descent directions of such functions. Examples wiil be given to demonstrate the performance of the algorithms.

■ FC-05

Friday, 10:15 - 12:15
23.3.9

Convex Analysis and Applications 1

Contributed session

Chair: *Edite M.G.P. Fernandes*, Production and Systems, University of Minho, School of Engineering, Campus de Gualtar, 4710-057, Braga, Portugal, emgpf@dps.uminho.pt

1 - Lipschitz modulus of the feasible set mapping for linear and convex semi-infinite systems under different perturbation settings

Juan Parra, Operations Research Center, Miguel Hernández University, Avda. del Ferrocarril s/n, (Edif. Torretamarit), 03202, Elche, Alicante, Spain, parra@umh.es, *Maria Josefa Cánovas*, *Francisco J. Gómez-Senent*

Our most particular setting is that of linear inequalities whose coefficients continuously depend on the index (ranging in a compact Hausdorff space), and right-hand-side perturbations. Our most general setting is that of convex inequalities (and possibly finitely many linear equations) with no particular functional dependence on the index (ranging on an arbitrary set), and where each constraint may be perturbed by means of an affine function. In all cases we provide (at least conceptually) computable expressions for the Lipschitz modulus relying only on the nominal system's data.

2 - A semi-infinite reduction type algorithm based on an exact penalty function

Alzira Mota, Mathematic, ISEP, Oporto, Portugal, atm@isep.ipp.pt, *A. Ismael F. Vaz*

During the last decades several algorithms were proposed for semi-infinite programming (SIP), but there is not much publicly available software. In this talk we propose a reduction type algorithm based on an exact penalty function for SIP. The proposed algorithm has been implemented in MATLAB and numerical results are shown with a set of test problems from the SIPAMPL database. The algorithm implementation is to be publicly available.

3 - Solving semi-infinite programming problems using filter method

Ana I. Pereira, Polytechnic Institute of Braganca, Campus de Sta Apolonia, Apartado 134, 5301-857, Braganca, Portugal, apereira@ipb.pt, *M. Fernanda P. Costa*, *Edite M.G.P. Fernandes*

Semi-infinite programming problems can be efficiently solved by reduction type methods. Here, we present a new global reduction method for Semi-infinite programming, where the multi-local optimization is carried out with a stretched simulated annealing algorithm, the reduced problem is approximately solved by a primal-dual interior point method combined with a three-dimensional filter line search strategy, and the global convergence is promoted through a two-dimensional filter line search. Numerical experiments with a set of well-known problems are shown.

4 - Regularity modulus of intersection mappings: Application to linear semi-infinite systems of equations and inequalities

Francisco J. Gómez-Senent, Operations Research Center, Miguel Hernández University, Orihuela, Alicante, Spain, paco.gomez@umh.es, *Maria Josefa Cánovas*, *Juan Parra*

The first part of this talk is devoted to relate the (metric) regularity modulus of the intersection mapping associated with a given finite family of set-valued mappings to the maximum of moduli of this family. We specifically refer to the so-called linear regularity and equirregularity properties. In the second part we determine the Lipschitz modulus of the feasible set mapping associated with a parameterized linear semi-infinite system containing a finite amount of equations, via the strategy of splitting them and applying results from linear inequality systems.

■ FC-06

Friday, 10:15 - 12:15
23.3.10

Conic and Semidefinite Programming 1

Contributed session

Chair: *Miguel Anjos*, Management Sciences, University of Waterloo, 200 University Avenue West, N2L 3G1, Waterloo, Ontario, Canada, anjos@stanfordalumni.org

1 - An Improved Characterisation of the Interior of the Completely Positive Cone

Peter Dickinson, Systems, Control and Applied Analysis, Johann Bernoulli Institute for Mathematics and Computer Science, University of Groningen, Bernoulliborg, P.O. Box 407, 9700 AK, Groningen, Groningen, Netherlands, P.J.C.Dickinson@rug.nl

The completely positive cone is the dual of the copositive cone and these cones are useful in binary and quadratic optimisation. Therefore it is important to study the properties of these cones. In this talk we present an improved characterisation of the interior of the completely positive cone in the form of a relaxation of the constraints from a previous characterisation. This is done by introducing the concept of the set of zeros in the nonnegative orthant for a quadratic form and looking at the properties of this set for copositive matrices.

2 - Approximating convex functions with linear, quadratic or semidefinite extended formulations

François Glineur, CORE, Université catholique de Louvain (UCLouvain), Voie du Roman Pays, 34, B-1348, Louvain-la-Neuve, Belgium, Francois.Glineur@uclouvain.be

An extended formulation expresses a convex set defined by large number of inequalities as a projection of a simpler (i.e. with less inequalities) but higher-dimensional set. This enables more efficient handling of the original set in optimization models.
We investigate the application of this technique to arbitrarily accurate approximations of several classes of convex functions (via their epigraphs), such as Euclidean norm, trigonometric functions, (matrix) logarithm, exponential and entropy, for which we present linear, quadratic or semidefinite extended formulations.

3 - Projection of a matrix onto the copositive cone

Julia Sponsel, Faculty of Mathematics and Natural Sciences, University of Groningen, Johann Bernoulli Institute for Mathematics and Computer Science, P.O. Box 407, 9700 AK, Groningen, Netherlands, J.K.Sponsel@rug.nl

In this talk we present an algorithm to approximate the projection of a matrix A onto the copositive cone C with arbitrary precision. The matrix is projected onto a sequence of polyhedral inner and outer approximations of C. We show that if the approximations converge to the copositive cone, the sequences of projections onto the inner resp. outer approximations converge to the projection of A onto C. Furthermore we go into the aspect how to construct the sequences of inner and outer approximations of C in order to improve the performance of the algorithm.

4 - Second-order relaxations of binary quadratic problems via polynomial programming

Miguel Anjos, Management Sciences, University of Waterloo, 200 University Avenue West, N2L 3G1, Waterloo, Ontario, Canada, anjos@stanfordalumni.org, *Bissan Ghaddar*, *Juan C. Vera*

We present a general framework for conic relaxations of binary quadratic programming based on polynomial programming. This framework provides not only stronger relaxations but also a cut-generation scheme to generate valid quadratic inequalities. Another advantage of the framework is that it permits to obtain second-order cone relaxations that are computationally more efficient than those based on general semidefinite programming. Computational tests show that our approach is competitive in terms of bounds and time when tested on quadratic knapsack problems.

Friday, 14:00 - 14:45

■ FD-01

Friday, 14:00 - 14:45
23.1.5

Invited Talk 2

Semi-plenary session

Chair: *Olga Kostyukova*, Mathematical System Theory, Institute of Mathematics, National Academy of Sciences of Belarus, Institute of Mathematics, Surganov Str.11,, 220072, Minsk, Belarus, kostyukova@im.bas-net.by

1 - A nasty cone with nice properties - new issues in copositive optimization

Immanuel Bomze, ISDS, University of Vienna, Bruenner Str. 72, A-1210, Vienna, Austria, immanuel.bomze@univie.ac.at

A symmetric matrix is called copositive, if it generates a quadratic form taking no negative values over the positive orthant. Contrasting to positive-semidefiniteness, checking copositivity is NP-hard. In a copositive optimization problem, we have to minimize a linear function of a symmetric matrix over the copositive cone subject to linear constraints. This convex program has no non-global local solutions. On the other hand, there are several hard non-convex programs which can be formulated as copositive programs, among them mixed-binary QPs or Standard QPs. Applications range from machine learning to several combinatorial optimization problems, including the maximum-clique problem or the maximum-cut problem.
The dual conic program, unlike the more popular SDP case, involves a different matrix cone, that of completely positive matrices (those which allow for a symmetric, possibly rectangular factorization with no negative entries). This conic optimization technique shifts complexity from global optimization towards sheer feasibility questions. Therefore it is of central importance to devise positive and negative certificates of copositivity and/or complete positivity. Three new copositivity tests based upon difference-of-convex (d.c.) decompositions are presented, and combined to a branch-and-bound algorithm of omega-subdivision type. The tests employ LP or convex QP techniques, but also can be used heuristically if educated guesses are available and preferred. We also propose some preprocessing ideas, which result in a normal form for copositivity. Switching to complete positivity, a heuristic factorization procedure for obtaining an explicit factorization is also presented.

■ FD-02

Friday, 14:00 - 14:45
23.1.6

Invited Talk 3

Semi-plenary session

Chair: *Domingos Cardoso*, Departamento de Matematica, Universidade de Aveiro, Campus Universitario de Santiago, 3810-193, Aveiro, Portugal, dcardoso@ua.pt

1 - Linear Optimization: Algorithms and Conjectures

Tamas Terlaky, Industrial and Systems Engineering, Lehigh University, H.G. Mohler Lab., 200 W. Packer Avenue, 18015, Bethlehem, Pennsylvania, United States, terlaky@lehigh.edu

In this talks we briefly review some fundamental algorithmic concepts for Linear optimization. The algorithms include elimination and pivot algorithms, ellipsoid and interior point methods and the perceptron algorithm. Complexity and convergence of the algorithms will be discussed. Open problems and conjectures related to both pivot algorithms and interior point methods will be discussed. Finally, we consider how the various algorithms could utilize the readily available multi-core architectures.

Friday, 15:00 - 16:30

■ FE-03

Friday, 15:00 - 16:30
23.3.4

Vector and Set-Valued Optimization I

Invited session

Chair: *Bienvenido Jiménez*, Departamento de Matemática Aplicada, UNED, Calle Juan del Rosal, 12, 28040, Madrid, Spain, bjimenez@ind.uned.es

Chair: *Ruben Lopez*, Departamento de Matematica y Fisica Aplicadas (DMFA), Universidad Catolica de la Santisima Concepcion, Alonso Ribera 2850, 409-0541, Concepcion, VIII Region, Chile, rlopez@ucsc.cl

1 - Strict maximality: stability, geometrical aspects and scalarization

Enrico Miglierina, Dipartimento di Economia, Università dell'Insubria, via Monte Generoso 71, 21100, varese, Italy, enrico.miglierina@uninsubria.it

The notion of strictly maximal point is a concept of proper maximality that plays an important role in the study of the stability of vector optimization problems. We give some on concerning stability properties for convex vector optimization problems and involving strictly maximal points. Moreover we study some properties of this notion with particular attention to geometrical aspects. More precisely, we individuate some relationships between strict maximality and the properties of the bases of the ordering cone. Finally, we deal with the linear scalarization for the strictly maximal points

2 - Solving a unification of vector optimization problems

Elvira Hernández, Matemática Aplicada, Universidad Nacional de Educación a Distancia, Juan del Rosal 12, 28040, Madrid, Spain, ehernandez@ind.uned.es, *Fabián Flores-Bazán*

In this talk, we analyze a general vector optimization problem in a unified framework via a nonlinear scalarizing function. Complete characterizations of the solution set to the vector problem are established and new optimality conditions are presented

3 - Existence of solutions in vector optimization problems via asymptotic analysis

Ruben Lopez, Departamento de Matematica y Fisica Aplicadas (DMFA), Universidad Catolica de la Santisima Concepcion, Alonso Ribera 2850, 409-0541, Concepcion, VIII Region, Chile, rlopez@ucsc.cl, *César Gutiérrez*, *Bienvenido Jiménez*, *Vicente Novo*

In this work, we revise some existence results for exact and approximate efficient and weak efficient solutions of vector optimization problems. By employing asymptotic analysis tools we prove that these existence results hold under weaker assumptions.

■ FE-04

Friday, 15:00 - 16:30
23.3.5

Multi Criteria Decision Making

Contributed session

Chair: *Ana Luisa Custodio*, Dept. Mathematics, New University of Lisbon, Quinta da Torre, 2829-516, Caparica, Portugal, alcustodio@fct.unl.pt

1 - Selecting the software that will be developed for the purchasing department using fuzzy AHP

Nurgül Demirtaş, Yildiz Technical University, 0090, Istanbul, Turkey, nurguldemirtas@gmail.com, *Özge Nalan Alp*, *Hayri Baraçlı*

Software applications are very important to follow operations in a company. These applications are influenced by many factors, when they are being developed. This situation is result in using multiple criteria decision making (MCDM) techniques under making a decision. In this study, firstly lacks of the software application which is used in purchasing department will be determined and then the most effective software application platform will be selected to developed applications by Fuzzy AHP which is the one of MCDM techniques.
Keywords: Fuzzy AHP, Software Selection,Decision Making

2 - Classical linear vector optimization duality revisited

Gert Wanka, Faculty of Mathematics, Chemnitz University of Technology, Chemnitz University of Technology, Faculty of Mathematics, D-09107, Chemnitz, Germany, gert.wanka@mathematik.tu-chemnitz.de, *Radu Ioan Bot*, *Sorin-Mihai Grad*

We consider the classical linear vector optimization problem and introduce a dual problem that has, different to some works in the literature which are of set-valued nature, a vector objective function. Weak, strong and converse duality results are proven and we also investigate its connections to other vector duals considered in the same framework in the literature (for an overview cf. R.I. Bot, S.-M. Grad, G. Wanka, Duality in vector optimization, Springer-Verlag, Berlin-Heidelberg, 2009). We also show that the efficient solutions coincide with the properly efficient solutions.

3 - Direct-Multisearch (DMS) for Multiobjective Optimization

Ana Luisa Custodio, Dept. Mathematics, New University of Lisbon, Quinta da Torre, 2829-516, Caparica, Portugal, alcustodio@fct.unl.pt, *Jose Aguilar Madeira*, *A. Ismael F. Vaz*, *Luís Nunes Vicente*

DMS is a novel derivative-free algorithm for multiobjective optimization, which does not aggregate any of the objective functions. Inspired by the search/poll paradigm of direct-search, DMS maintains a list of nondominated points, from which the new iterates or poll centers are chosen.
Under common assumptions, it is proved that at least one limit point of the sequence of iterates lies in the Pareto frontier. Computational results are reported, which show that DMS has an impressive capability of generating the whole Pareto frontier even without using a search step.

■ FE-05

Friday, 15:00 - 16:30
23.3.9

Convex Analysis and Applications 2

Contributed session

Chair: *Juan Parra*, Operations Research Center, Miguel Hernández University, Avda. del Ferrocarril s/n, (Edif. Torretamarit), 03202, Elche, Alicante, Spain, parra@umh.es

1 - A primal-dual approach to argmin and subdifferential calculus

Marco A. López-Cerdá, Statistics and Operations Research, Alicante University, Ctra. San Vicente de Raspeig s/n, 3071, Alicante, Spain, marco.antonio@ua.es

We provide a primal-dual approach to the characterization of optimal sets and subdifferential calculus. The main results presented in this talk are linked with some recent results in [1], [2]. [1] J.-B.Hiriart-Urruty, M.A.Lopez, M.Volle "The epsilon-strategy in variational analysis: illustration with the closed convexification of a function", to appear in Rev. Mat. Iberoamericana. [2] M.A.Lopez, M.Volle "A formula for the set of optimal solutions of a relaxed minimization problem. Applications to subdifferential calculus"to appear in Journal of Convex Analysis, 17, 2010.

2 - Interior point methods for sufficient complementarity problems

Florian Potra, Mathematics & Statistics, University of Maryland, MD 21250 , Baltimore, United States, potra@umbc.edu

We present three interior point methods for sufficient horizontal linear complementarity problems (SHLCP): a large update method, a first order corrector-predictor, and a second order corrector-predictor. They use the wide neighborhood of the central path introduced by Ai and Zhang. The methods do not depend on the handicap of the SHLCP, and they have the best known iteration complexity for SHLCP. The second method is Q-quadratically convergent under the strict complementarity assumption, while the third method is Q-superlinearly convergent even without this assumption.

3 - Pointbased Characterizations of Calmness and Subregularity of Constraint Set Mappings

Helmut Gfrerer, Institute for Computational Mathematics, Johannes Kepler University Linz, Altenbergerstr 69, A-4040, Linz, Austria, gfrerer@numa.uni-linz.ac.at

The existence of nondegenerate multipliers in first-order necessary conditions at a local minimizer is related to the validity of some constraint qualification condition, for instance the property of metric subregularity of the constraint set mapping or equivalently, the calmness property of the solution mapping. In this talk we present characterizations of calmness and subregularity, respectively. We will see that there are some limitations when using exclusively first-order analysis, which can be bypassed assuming some part of the constraint mapping to be known subregular.

■ FE-06

Friday, 15:00 - 16:30
23.3.10

Conic and Semidefinite Programming 2

Contributed session

Chair: *Adelaide Cerveira*, DM & CIO, UTAD, 5000, Vila Real, adelaide.cerveira@sapo.pt

1 - On the Accuracy of Uniform Polyhedral Approximations of the Copositive Cone

E. Alper Yildirim, Department of Industrial Engineering, Bilkent University, Bilkent University, Department of Industrial Engineering, Bilkent, 06800, Ankara, Turkey, yildirim@bilkent.edu.tr

We consider linear optimization problems over the cone of copositive matrices (copositive programs). We propose a hierarchy of increasingly better outer polyhedral approximations to the copositive cone. By combining this hierarchy with the inner polyhedral approximations of de Klerk and Pasechnik, we obtain a sequence of increasingly sharper lower and upper bounds on the optimal value of a copositive program, both of which are exact in the limit under mild assumptions. For certain classes of optimization problems, we derive tight bounds on the gap between the upper and lower bounds.

2 - Robust Nash equilibria in incomplete information games: semidefinite complementarity reformulation and equilibrium behavior

Shunsuke Hayashi, Graduate School of Informatics, Kyoto University, Yoshida-Honmachi, Sakyo-Ku, 606-8501, Kyoto, Japan, shunhaya@amp.i.kyoto-u.ac.jp, *Ryoichi Nishimura*, *Masao Fukushima*

The robust Nash equilibrium results from each player's decision-making based on the robust optimization policy. In this study, we focus on the N-person non-cooperative game in which each player's cost function parameters and the opponents' strategies are uncertain in the sense of Euclidean norm. Moreover we show that the robust Nash equilibrium problem can be reformulated as a semidefinite complementarity problem (SDCP), by utilizing the duality theory for nonconvex quadratic programming. We also give some numerical results that show interesting behavior of obtained robust Nash equilibria.

3 - Avoiding free bars in a branch and bound frame work to solve truss topology design problems with discrete areas

Adelaide Cerveira, DM & CIO, UTAD, 5000, Vila Real, adelaide.cerveira@sapo.pt, *Agostinho Agra*, *Fernando Bastos*, *Joaquim Gromicho*

One of the classical problems in the structural optimization field is to find the stiffest truss, under a given load and with a bound on the total volume. The topology of the truss is generated by varying the cross-sectional areas of the bars, allowing zero values. In this work we consider a finite set of possible values for those cross sectional areas. We propose a Semidefinite Programming formulation with discrete variables. In order to solve the problem we derive and compare two branch and bound algorithms.

Friday, 17:00 - 18:30

■ FF-03

Friday, 17:00 - 18:30
23.3.4

Vector and Set-Valued Optimization II

Invited session

Chair: *Bienvenido Jiménez*, Departamento de Matemática Aplicada, UNED, Calle Juan del Rosal, 12, 28040, Madrid, Spain, bjimenez@ind.uned.es

Chair: *Elvira Hernández*, Matemática Aplicada, Universidad Nacional de Educación a Distancia, Juan del Rosal 12, 28040, Madrid, Spain, ehernandez@ind.uned.es

1 - New conditions for strict local minima of order m in constrained multiobjective optimization

Marcin Studniarski, Faculty of Mathematics and Computer Science, University of Lodz, ul. S. Banacha 22, 90-238, Lodz, marstud@math.uni.lodz.pl

We present some new necessary and sufficient conditions for strict local Pareto minima in a constrained multiobjective optimization problem. They are formulated in terms of lower and upper m-th order directional derivatives of vector-valued functions. A comparison with the results of Do Van Luu (Optimization 57, 2008, 593-605) is included.

2 - Scalarization and well-posedness for set-valued optimization

Elena Molho, Dipartimento di Economia Politica e Metodi Quantitativi, Università di Pavia, Via San Felice 5, 27100, Pavia, Italy, molhoe@eco.unipv.it

We consider the set optimization approach where the ordering cone in the image space induces an ordering on the images of the set-valued objective map. We develop a scalarization scheme directly based on the order relation on the space of the image sets. We define a notion of well-posedness for set optimization based on the notion of strict minimizer and we prove the equivalence between the well-posedness of the original set optimization problem and the Tykhonov well-posedness of the scalarized problem.

3 - A smooth vector variational principle

Ewa Bednarczuk, Modelling and Optimization of Dynamical Systems, Systems Research Institute, Warsaw, Poland, bednarcz@ibspan.waw.pl

For mappings f taking values in a vector space Y, the existing vector variational principles are directional in the sense that for a given direction e from Y, a real-valued perturbation function g is constructed such that the mapping f+ge has an efficient point. We provide a smooth vector variational principle for directions of perturbations belonging to a closed convex subset D of a reflexive Banach space Y. The talk is based on the joint work with Dariusz Zagrodny.

■ FF-04

Friday, 17:00 - 18:30
23.3.5

Global Optimization 1

Contributed session

Chair: *Eligius M.T. Hendrix*, Computer Architecture, Universidad de Málaga, Campus de Teatinos, ETSI 2.2.28, 29017, Malaga, Spain, eligius.hendrix@wur.nl

1 - Solution of the Asymmetric Eigenvalue Complementarity Problem

Silvério Rosa, Department of Mathematics, University of Beira Interior and Institute of Telecommunications, Rua Marquês d'Ávila e Bolama, n. 1, 6201-01, Covilhã, Portugal, rosa@ubi.pt, *Joaquim Judice*

Given a real matrix A and a real Positive Definite matrix B, the Eigenvalue Complementarity Problem (EiCP) is an extension of the Generalized Eigenvalue Problem where the variables are non-negative and satisfy a complementarity constraint. The asymmetric EiCP deals with the case where at least A or B is asymmetric and reduces to a Variational Inequality and to a Global Optimization Problem. A projected gap-function method and an enumerative algorithm are introduced for finding a solution to the asymmetric EiCP. The computation of several complementary eigenvalues and of the maximum/minimum of these eigenvalues is also studied

2 - Optimization of the reflection coefficient for the absorbing boundary conditions for the linear Schrodinger equation

Julius Zilinskas, Systems Analysis Department, Institute of Mathematics and Informatics, Akademijos 4, LT 08663, Vilnius, Lithuania, zilinskasjulius@gmail.com, *Raimondas Ciegis*

In order to solve the linear Schrodinger equation in an unbounded domain the computation domain should be restricted and artificial absorbing boundary conditions should be formulated. The exact boundary conditions are nonlocal in time, therefore they are not efficient in numerical simulations. We consider an approximation of the reflection coefficient by the rational function, which parameters are obtained using global optimization. Results of computational experiments are presented and the dependence of the accuracy of absorbing boundary conditions on the number of parameters is investigated.

3 - On Robustness in continuous facility location

Eligius M.T. Hendrix, Computer Architecture, Universidad de Málaga, Campus de Teatinos, ETSI 2.2.28, 29017, Malaga, Spain, eligius.hendrix@wur.nl, *Rafael Blanquero*, *Emilio Carrizosa*

A new continuous location model is presented and embedded in the literature on robustness in facility location. The multimodality of the model is investigated, and a branch and bound method based on dc optimization is described. Numerical experiments are reported, showing that the developed method allows one to solve problems with thousands of demand points in a few seconds.

■ FF-05

Friday, 17:00 - 18:30

23.3.9

Convex Analysis and Applications 3

Contributed session

Chair: *Yurii Nesterov*, CORE, Université catholique de Louvain (UCL), 34 voie du Roman Pays, 1348, Louvain-la-neuve, Belgium, Yurii.Nesterov@uclouvain.be

1 - On the weak* closedness of the sum of the images of enlargements of two maximal monotone operators

Radu Ioan Bot, Faculty of Mathematics, Chemnitz University of Technology, Reichenhainer Str. 39 Zi. 612, 09107, Chemnitz, Germany, bot@mathematik.tu-chemnitz.de, *Ernö Robert Csetnek*

In this talk we deal with a general class of enlargements of monotone operators defined on general Banach spaces, introduced by means of the representative functions of the operators in discussion. By making use of some convex analysis techniques, we show that, under the fulfillment of a weak interior-point regularity condition, the sum of the images of enlargements of two maximal monotone operators is weak* closed. In this way we improve some results recently given in the literature.

2 - Double Smoothing Algorithm for Infinite-dimensional Optimization problems with coupling and pointwise constraints.

Olivier Devolder, CORE, Université catholique de Louvain (UCL), Louvain-la-Neuve, Belgium, olivier.devolder@uclouvain.be, *François Glineur*, *Yurii Nesterov*

We consider convex infinite-dimensional problems in a functional space with pointwise constraints and a finite number of linear coupling constraints. In order to solve this problem class, we follow a dual approach that does not rely on preliminary discretization. Dualizing the coupling constraints and applying a double smoothing to the dual function, we can use optimal schemes of finite-dimensional smooth convex optimization. Using the generated dual sequence, we reconstruct a nearly optimal and feasible primal solution. Our strategy is supported by a worst-case complexity analysis.

3 - Characterizing surjectivity results for the sum of two maximal monotone operators with representative functions

Sorin-Mihai Grad, Faculty of Mathematics, Chemnitz University of Technology, 09107, Chemnitz, Sachsen, Germany, grad@mathematik.tu-chemnitz.de, *Radu Ioan Bot*

We characterize some surjectivity results concerning the sum of two maximal monotone operators by using representative functions. Weak regularity conditions that guarantee these results are also provided and some applications are presented.

■ FF-06

Friday, 17:00 - 18:30

23.3.10

Nonsmooth Optimization 1

Contributed session

Chair: *Leonidas Sakalauskas*, Statistical MOdelling, Institute of Mathematics&Informatics, Akademijos 4, 2600, Vilnius, Lithuania, sakal@ktl.mii.lt

1 - Sparse Reconstruction of First and Second Order Information and its Application to Derivative-Free Optimization

Afonso Bandeira, Matematica, Universidade de Coimbra, Coimbra, Portugal, afonsobandeira@gmail.com, *Katya Scheinberg*, *Luís Nunes Vicente*

Trust-region interpolation-based methods are a popular class of methods for Derivative-Free Optimization which relies on locally minimizing quadratic models formed frequently from underdetermined interpolation. On the other hand, recent advances in the field of compressed sensing provide conditions under which a sparse signal can be accurately recovered from very few random measurements, by minimizing the l1-norm of the unknown signal. We explore an approach to built the model by minimizing the l1-norm of the model Hessian. The preliminary results from this technique are very encouraging.

2 - A Rank-Based Fitness Artificial Fish Swarm Algorithm for Global Optimization

Ana Maria A.C. Rocha, Production and Systems, University of Minho, Campus de Gualtar, 4710-057, Braga, Portugal, arocha@dps.uminho.pt, *Edite M.G.P. Fernandes*, *Tiago F. M. C. Martins*, *Joana P. Fernandes*

Nonlinear programming problems are known to be difficult to solve, especially when the objective function is multimodal. Optimization algorithms based on swarm intelligence have been used with success. The artificial fish swarm algorithm is a population-based technique that simulates fish swarm behaviors. To solve nonlinear constrained problems, a constraint handling technique based on ranking the points with respect to the objective function and constraint violation independently is proposed. Numerical results are presented to assess the algorithm performance.

3 - ϵ-Feasilble Interior-Point Method for two stage stochastic programming

Leonidas Sakalauskas, Statistical MOdelling, Institute of Mathematics&Informatics, Akademijos 4, 2600, Vilnius, Lithuania, sakal@ktl.mii.lt

A stochastic adaptive method has been developed to solve stochastic linear problems by a finite sequence of Monte-Carlo sampling estimators. The method is based on the adaptive regulation of the size of Monte-Carlo samples and a statistical termination procedure taking into consideration statistical modelling accuracy. Our approach distinguishes itself by the treatment of accuracy of the solution in a statistical manner, testing the hypothesis of optimality according to statistical criteria, and estimating confidence intervals of the objective and constraint functions. To avoid "jamming' or "zigzagging' solving a constraint problem t he interior-point method is implemented to establish the ϵ—feasible direction. The proposed adjustment of a sample size, when it is taken inversely proportional to the square of the norm of the Monte-Carlo estimate of the gradient, guarantees convergence a. s. at a linear rate. The numerical study and examples in practice corroborate theoretical conclusions and show that the developed procedures make it possible to solve stochastic problems with sufficient accuracy by the means of an acceptable size of computations.

Saturday, 9:00 - 9:45

■ SA-01

Saturday, 9:00 - 9:45
23.1.5

Invited Talk 4

Plenary session

Chair: *Mirjam Duer*, Johann Bernoulli Institute of Mathematics and Computer Science, Rijksuniversiteit Groningen, P.O. Box 407, 9700 AK, Groningen, Netherlands, M.E.Dur@rug.nl

1 - Direct Search for Single and Multiobjective Optimization

Luís Nunes Vicente, University of Coimbra, 3012, Coimbra, Portugal, lnv@mat.uc.pt

Direct search is a popular class of algorithms for the minimization of a function without derivatives. Most direct-search methods are based on evaluating the objective function at a set of (poll) points defined by a positive spanning set.
In this talk we will characterize the behavior of direct-search methods under adverse conditions by (i) studying its global convergence properties and numerical behavior when the objective function is discontinuous and (ii) presenting a worst case complexity bound on the number of function evaluations.
Then, we will briefly introduce an extension of direct search to multiobjective optimization (called direct multisearch) by essentially changing the polling paradigm to incorporate nondominancy tests. Direct multisearch enjoys rigorous global convergence properties and exhibits excellent numerical performance (as it will be shown in the contributed talk by A. L. Custódio).

Saturday, 10:15 - 11:00

■ SB-01

Saturday, 10:15 - 11:00
23.1.5

Special Lecture on Hirsh's Conjecture

Semi-plenary session

Chair: *Tamas Terlaky*, Industrial and Systems Engineering, Lehigh University, H.G. Mohler Lab., 200 W. Packer Avenue, 18015, Bethlehem, Pennsylvania, United States, terlaky@lehigh.edu

1 - A counter-example to the Hirsch conjecture

Francisco Santos Leal, Matemáticas, Estadística y Computación, University of Cantabria, Facultad de Ciencias,, Avda De Los Castros S/n, 39005, Santander, Cantabria, Spain, francisco.santos@unican.es

The Hirsch conjecture, stated in 1957, said that if a polyhedron is defined by n linear inequalities in d variables then its combinatorial diameter should be at most $n - d$. That is, it should be possible to travel from any vertex to any other vertex in at most $n - d$ steps (traversing an edge at each step). The unbounded case was disproved by Klee and Walkup in 1967. In this talk I will describe the construction of the first counter-example to the bounded case (a polytope).
The conjecture was posed and is relevant in the context of the simplex method in linear programming. The simplex method, after all, finds the optimal solution by moving from vertex to vertex along the edges of the feasibility polyhedron. Experimentally, the simplex method usually finishes in a linear number of steps, but examples where certain choices of pivot rules lead to exponentially long paths exist, and no pivot rule is known that can be proved to always finishes in polynomial number of steps. Although other methods for linear programming are proved to be polynomial (Karmakar, Khachiyan), the simplex method remains one of the methods most often used in practice.
From a complexity theory point of view, it is also significant that the known methods are polynomial in the "bit complexity"model, but a polynomial pivot rule for the simplex method would provide a "strongly polynomial"algorithm, that is, one that is polynomial also in the "real machine"model. The question whether such a "strongly polynomial"method exists for linear programming was included by S. Smale in his list of "Mathematical problems for the next century"(AMS, 2000). Of course, a polynomial pivot rule can only exist if the combinatorial diameter is polynomially bounded.

Saturday, 10:15 - 12:15

■ SC-03

Saturday, 10:15 - 12:15
23.3.4

Nonlinear and nonsmooth optimization and applications (in memory of Prof. Mikhail Mikhalevich)

Invited session

Chair: *Tatiana Tchemisova*, Departmento of Mathematics, University of Aveiro, Campus Universitario de Santiago, 3810-193, Aveiro, Portugal, tatiana@ua.pt

Chair: *Ludmilla Koshlai*, Systems analysis and OR, Institute of Cybernetics, Gonchar str.,65-a,apt.20, 01054, Kiev, Ukraine, koshlai@ukr.net

1 - Identification of coefficients for nonlinear heat equation

Valentin Borukhov, Mathematical system theory, Institute of mathematics, Belarussian National Academy of Sciences, Institute of Mathematics, Surganov Str.11,, 220072, Minsk, Belarus, borukhov@im.bas-net.by, *Olga Kostyukova*, *Mariya Kurdina*

We consider the inverse problems of reconstructing the thermal-conductivity coefficients and heat exchange coefficients for nonlinear time-dependent heat equations. For determination of the nonlinear thermal-conductivity coefficients, we propose a new functional identification approach. The approach is based on a gradient-type method for cost functional minimization. Algorithm for heat exchange coefficient reconstruction is based on a method of stage-by-stage optimization. Total variation regularization is used for noise filtering. Results of numerical experiments are presenter.

2 - Application of Nonlinear Optimization to the Problems of Structural Changes during Transition

Ludmilla Koshlai, Systems analysis and OR, Institute of Cybernetics, Gonchar str.,65-a,apt.20, 01054, Kiev, Ukraine, koshlai@ukr.net, *Petro Stetsyuk*

We consider optimization models for improvement of the existed and implementation of new technologies necessary for Ukrainian economy. The input matrix of export and import values as well as the share of labour cost changes are model variables. Multiply criteria, resources and ecological constraints are taken into account in the models, thus, they born complicated nonlinear optimization problems. The efficient subgradient method has been proposed to solve them. Models, methods and databases are integrated into specialized DSS, which is designed as open menu-driven software.

3 - Analysis of the Interdependence of Market Indices

Tamara Bardadym, Dept. of Nonsmooth Optimization Methods, V.M.Glushkov Institute of Cybernetics, Prospekt Akademika Glushkova, 40, 03187, Kyiv, Ukraine, TBardadym@gmail.com, *Jean-Francois Emmenegger*, *Elena Pervukhina*

A cointegration approach for pair wise comparisons of popular financial indices is presented. Particularly, the dynamic changes of the PFTS stock exchange, the larger of both Ukrainian two main indices and the RTS, the Russian stock market index and some other financial indices are studied. The estimated cointegration models show national peculiarities of the contemporary economic transition processes. The paper is supported by SNSF, SCOPES-project Nr: IZ73ZO_127962.

4 - Decision Support System for Planning Structural and Technological Changes

Petro Stetsyuk, Nonsmooth optimization methods, Institute of Cybernetics, 40 Prospect Glushkova, 03187, Kiev, stetsyukp@gmail.com, *Ludmilla Koshlai*, *Oleksandr Pylypovskyi*

Model-driven DSS for decision support during the structural reforms in post-communist economies is considered. Optimization models for improvement of the existed and implementation of new technologies together with models of search of rational export-import structure create the core of this system. A specific feature of the user interface is presence of two types of windows in its structure: for a user being an expert in economy and for an expert in optimization methods. The protocol of interaction with user is automatically recorded in html format under the DSS control.

5 - On Stochastic Optimization Techniques to Analyze Time Series of the Ukrainian Tracking Industry

Elena Pervukhina, Dept. of Technical Mechanics and Mashines, Sevastopol National Technical University, Universitetskaya Str., 33, 99053, Sevastopol, elena@pervuh.sebastopol.ua, *Jean-Francois Emmenegger*

Some optimization techniques in empirical modeling are applied to summarize the data describing the development of the Ukrainian tracking industry. The aim of this paper is to understand the empirical evidence and to augment and consolidate the knowledge about the methods, how the Ukrainian tracking industry works compared to the tracking industry of developed countries. Stochastic optimization techniques are applied to smooth the data and to forecast future cargo volumes.

■ SC-04

Saturday, 10:15 - 12:15
23.3.5

Global Optimization 2

Contributed session

Chair: *Alexander Plakhov*, Department of Mathematics, University of Aveiro, Campus Santiago, 3810-276, Aveiro, Portugal, plakhov@ua.pt

1 - Optimizing Rotation of Connected Circles

Uwe Nowak, Optimization, Fraunhofer ITWM, Fraunhofer-Platz 1, 67663, Kaiserslautern, Germany, uwe.nowak@itwm.fraunhofer.de, *Karl-Heinz Küfer*

For a set of fixed centered, rotatable circles with pins in their interior we minimize the weighted pin connection length in squared Euclidean distance. By analyzing two different but equivalent connection models (clique and star) we give a convex relaxation and show a nonlinear optimization algorithm with absolute performance guarantee. Finally we show the special case of a single net. This is equivalent to the facility location problem of placing a single facility with minimal distance sum to circle perimeter shaped demand areas.

2 - Lipschitz global optimization and local information

Dmitri Kvasov, Department of Electronics, Computer Science and Systems, University of Calabria, DEIS, Via P. Bucci, Cubo 42C, I-87036, Rende (CS), Italy, kvadim@si.deis.unical.it, *Yaroslav Sergeyev*

Global optimization problems with black-box multiextremal objective functions are considered. Solving them efficiently is a great challenge since they present a high number of local minima, often with extremely different function values, and do not present a simple mathematical description of the global optima. In this occasion, considering the only global information about behavior of the function during its optimization can lead to a slow convergence of algorithms to global optimum points. Therefore, various aspects of the local information usage in global optimization methods are discussed.

3 - Locating median lines in three dimensional space

Emilio Carrizosa, Estadistica e Investigacion Operativa, Universidad de Sevilla, Matematicas, Reina Mercedes s/n, 41012, Sevilla, Spain, Spain, ecarrizosa@us.es, *Rafael Blanquero*, *Anita Schoebel*, *Daniel Scholz*

Given a set of points in the three-dimensional Euclidean space, we address the problem of locating the line minimizing the sum of the distances to such points to the line.
Structural properties of this global optimization problem are discussed, and an algorithmic approach to obtain a global optimum is presented. The computational experience reported shows that the problem is much harder than its 2-d version.

4 - Problems of minimal and maximal resistance in billiards

Alexander Plakhov, Department of Mathematics, University of Aveiro, Campus Santiago, 3810-276, Aveiro, Portugal, plakhov@ua.pt

A parallel flow of non-interacting particles with unit velocity falls on a bounded body. Some particles hit the body, making one or more elastic reflections from its boundary, and thus impart to it some momentum. As a result, a force of aerodynamic resistance with respect to a given direction is created. We consider problems of optimizing the mean value of resistance relative to a given set of directions. We present bodies of zero resistance in one and two directions. Also, we construct sequences of bodies maximizing the mean resistance relative to all directions on the unit sphere.

■ SC-05

Saturday, 10:15 - 12:15
23.3.9

Real World Problems

Contributed session

Chair: *Gerhard-Wilhelm Weber*, Institute of Applied Mathematics, Middle East Technical University, ODTÜ, 06531, Ankara, Turkey, gweber@metu.edu.tr

1 - Optimal control by queueing systems with broadcasting service

Alexander Dudin, Applied Mathematics and Computer Science, Belarusian State University, 4, Independence Ave., Minsk-30, 220030, Minsk, Belarus, dudin@bsu.by, *Valentina Klimenok*, *Chesoong Kim*, *Bin Sun*

We consider a multi-server queueing system with the Markov Arrival Process and phase type service time distribution where a customer is served by several servers if the number of free servers is not too large and not too small. We show advantages of this service discipline in comparison to the classical service discipline such as smaller average sojourn time and higher probability of the correct service. Problem of the optimal choice of the limiting number of servers, which can be involved into the service of an arbitrary customer, is numerically solved. Illustrative examples are presented.

2 - Fish Population Dynamics and Modeling

Dmitriy Stukalin, Greifswald University, Germany, dimidron85@rambler.ru

Issues related to the implementation of dynamic programming for optimal control of a three-dimensional dynamic model, such as the fish populations management problem are presented. They belong to a class of models called Lotka-Volterra Models.
The existence of steady states and their stability is studied using eigenvalue analysis. Boundedness of the exploited system is examined. The existence of bionomic equilibria will be considered. The problem of optimal harvest policy is then solved by using Pontryagin's maximum principle.

3 - A study of constrained optimization methods for joint inversion of geophysical datasets

Leticia Velazquez, Mathematical Sciences, The University of Texas at El Paso, 500 West University Avenue, 79968-0514, El Paso, Texas, United States, leti@utep.edu, *Aaron Velasco*, *Anibal Sosa*, *Miguel Argaez*, *Rodrigo Romero*

Inverse problems arise in geophysical applications when characterizing the earth structure using seismic data. We focus on the least squares joint inversion of different datasets from the same region. We conduct a study of regularization techniques and Primal-Dual Interior-Point methods for incorporating restrictions in the parameterized subsurface layered velocities. We present a numerical experimentation using synthetic data from teleseismic P-wave receiver functions and surface wave dispersion velocities, and conclude which strategy characterize better a region.

4 - On Identification and Optimization of Financial Processes

Gerhard-Wilhelm Weber, Institute of Applied Mathematics, Middle East Technical University, ODTÜ, 06531, Ankara, Turkey, gweber@metu.edu.tr

In this talk, we firstly investigate problems of structure and identification of stochastic differential equations and of their time-discrete varities. We employ our newly developed tools GAM&CQP, GPLM&CQP, CMARS, Robust CMARS, and nonlinear regression. Then we discuss the corresponding finanical processes, and apply advanced portfolio optimization on them. We finally relate this with risk management, new classification tools and uncertainty handling, demonstrating how modern continuous optimization becomes a key technology here.

■ SC-06

Saturday, 10:15 - 12:15
23.3.10

Variational Calculus and Optimal Control

Invited session

Chair: *Delfim F. M. Torres*, Department of Mathematics, University of Aveiro, Universidade de Aveiro, Campus Universitário de Santiago, 3810-193 , Aveiro, Portugal, delfim@ua.pt

Chair: *Moulay Rchid Sidi Ammi*, Department of Mathematics, Faculté des Sciences et Techniques Errachidia, Morocco, B.P 509 Boutalamine, 52000, Errachidia, Morocco, sidiammi@ua.pt

1 - Higher-order Hahn's quantum variational calculus

Artur M. C. Brito da Cruz, University of Aveiro, 3800, Aveiro, artur.cruz@estsetubal.ips.pt, *Natália Martins*, *Delfim F. M. Torres*

We prove a necessary optimality condition of Euler-Lagrange type for quantum variational problems involving Hahn derivatives of higher-order. As particular cases, we obtain the q-calculus and h-calculus higher-order Euler-Lagrange equations. We end with a simple example of a quantum optimization problem where our results lead to the global maximizer, which is not a continuous function, while the previous methods available in the literature fail to apply.

2 - Application of an approximation method to variational problems with fractional derivatives

Shakoor Pooseh, Department of Mathematics, University of Aveiro, Campus de Santiago, Universidade de Aveiro, 3810-193, Aveiro, Aveiro, Portugal, spooseh@ua.pt, *Delfim F. M. Torres*

Recently it has been shown that a variational problem with fractional derivatives can be reduced to a classical problem using an approximation of the Riemann-Liouville fractional derivatives in terms of a finite sum where only derivatives of integer order are present. In this work we examine the validity of such approximations on some well known classical problems. We substitute the first order derivative with a fractional one and study the approximation by considering the fractional order close enough to one.

3 - Necessary optimality conditions for fractional variational problems

Tatiana Odzijewicz, Mathematics, University of Aveiro, Rua Mario Sacramento, BL 53 3 303, 3810-106, Aveiro, Portugal, tatianao@ua.pt, *Delfim F. M. Torres*

We give a proper fractional extension of the classical calculus of variations. Necessary optimality conditions of Euler-Lagrange type for variational problems containing both fractional and classical derivatives are proved. The fundamental problem of the calculus of variations with mixed integer and fractional order derivatives as well as isoperimetric problems are considered.

4 - Conjugate points for minimal time bang-bang control problems

Cristiana Silva, Mathematic, University of Aveiro, Rua Casal Cantiga, 16, Cela de Cima, 2440-158 , Batalha, Portugal, cjoaosilva@ua.pt

We propose a regularization procedure that allows to use algorithms for the computation of conjugate times in the smooth case on the computation of conjugate times of minimal time control problems whose optimal controls are bang-bang.

Saturday, 14:00 - 14:45

■ SD-01

Saturday, 14:00 - 14:45
23.1.5

Invited Talk 5

Semi-plenary session

Chair: *Florian Potra*, Mathematics & Statistics, University of Maryland, MD 21250 , Baltimore, United States, potra@umbc.edu

1 - Risk averse stochastic programming

Alexander Shapiro, Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA30332, 30332, Atlanta, Georgia, United States, ashapiro@isye.gatech.edu

In recent years a theory of the so-called coherent risk measures had emerged. Its application to static (two-stage) stochastic programming is well established and generally accepted by the scientific community. The multistage case, on the other hand, is much more delicate. In this talk we discuss various approaches to multistage risk averse stochastic programming and point to involved conceptual and numerical difficulties.

■ SD-02

Saturday, 14:00 - 14:45
23.1.6

Invited Talk 6

Semi-plenary session

Chair: *Emilio Carrizosa*, Estadistica e Investigacion Operativa, Universidad de Sevilla, Matematicas, Reina Mercedes s/n, 41012, Sevilla, Spain, Spain, ecarrizosa@us.es

1 - Optimally Fitting Hyperplanes to Data

Mirjam Duer, Johann Bernoulli Institute of Mathematics and Computer Science, Rijksuniversiteit Groningen, P.O. Box 407, 9700 AK, Groningen, Netherlands, M.E.Dur@rug.nl

Suppose you are interested in estimating the slope of a line fitted to data points. How should you fit the line if you want to treat each variable on the same basis?
Least squares regression is inappropriate here since it's putpose is the predictions of one of the variables, so it requires to specify which variables are the independent variables and which one is the dependent variable. A change in this setting will lead to a completely different least squares estimate. Moreover, the least squares model is based on the assumption that only the dependent variable is subject to measurement errors; the independent variables are assumed to be known exactly, a premise that is often not fulfilled.
We will present an approach that avoids these shortcomings. The basic idea is that a different criterion is chosen to be minimized for the optimal hyperplane: instead of minimizing the sum of the squares of the residuals, we consider the deviations for each variable and multiply them. The new estimate possesses desirable theoretical properties and comes with a nice geometrical interpretation. However, in contrast to least squares regression it requires the solution of a global optimization problem.

Saturday, 15:00 - 17:00

■ SE-03

Saturday, 15:00 - 17:00
23.3.4

Linear and Convex Programming

Contributed session

Chair: *Carlos J. Luz*, Math, Escola Sup. Tecnologia Setúbal / Instituto Politécnico de Setúbal, Campus do IPS, Estefanilha, 2910-761, Setúbal, Portugal, carlos.luz@estsetubal.ips.pt

1 - Inverse problem of linear programming with minimum variation in right hand side vector

Mohammad Reza Peyghami, Mathematics, K.N. Toosi University of Technology, Math. Department, K.N. Toosi University of Technology,, P.O.Box 16315-1618, Tehran, Iran, 16315-1618, Tehran, Tehran, Iran, Islamic Republic Of, peyghami@kntu.ac.ir

Inverse linear programming problems have very practical applications in the real world problems. In this talk, we first introduce the inverse problem of linear programming in which we would like to adjust the right hand side as less as possible so that the current operating plan becomes optimal, and then we present an algorithm based on interior point methods structure to solve it. The algorithm has also been tested on some various problems.

2 - A computational approach to the central path of a redundant Klee-Minty cube

Kees Roos, EWI, TU Delft, Mekelweg 4, 2628CD, Delft, ZH, Netherlands, c.roos@tudelft.nl, *B. P. Silalahi*

In 1972 Klee and Minty introduced a Linear Optimization problem whose domain is a perturbed n-dimensional unit cube. Recently (Deza, Nematolahi, Peyghami, Terlaky, 2006) have shown that when adding redundant constraints to the Klee-Minty problem then the central path may visit small neighborhoods of all these vertices. They obtained a new tight lower bound for the number of iterations required by any IPM. We present another redundant Klee-Minty problem leading to the same lower bound, and also propose a computational model that provides some numerical evidence that it cannot be improved.

3 - An optimisation model for urban air pollution control utilising climate data

Sven-AAke Gustafson, University of Stavanger, Stavanger, 4036, Stavanger, sven4014@yahoo.no

We consider the problem of maintaining good air quality in an urban area while minimising control costs. The input to our model consists of climate data and a list of source classes for the pollutants under study as well as air quality standards. These are formulated as quantitative conditions at each receptor point. Thus we get an optimisation problem with finitely many decision variables and infinitely many constraints, since these are associated with each receptor point. An optimal solution may be calculated by means of algorithms of semi-infinite programming.

4 - Upper bounds based on quadratic programming for the maximum size of a k-regular induced subgraph

Carlos J. Luz, Math, Escola Sup. Tecnologia Setúbal / Instituto Politécnico de Setúbal, Campus do IPS, Estefanilha, 2910-761, Setúbal, Portugal, carlos.luz@estsetubal.ips.pt

This talk deals with the NP-hard problem of determining in a graph a maximum cardinality subset of vertices inducing a k-regular subgraph. Its aim is to disclose three upper bounds based on quadratic programming recently obtained for the optimum value of the above problem. The first one is derived from a well known result of Motzkin and Straus. The second is an improvement of an upper bound previously introduced in Cardoso, Kaminski, and Lozin. Finally, the third bound follows from the existing connection between the k-regular induced subgraph problem and the co-k-plex problem.

■ SE-04

Saturday, 15:00 - 17:00
23.3.5

Global Optimization 3

Contributed session

Chair: *Immanuel Bomze*, ISDS, University of Vienna, Bruenner Str. 72, A-1210, Vienna, Austria, immanuel.bomze@univie.ac.at

1 - Weighting coefficients versus constraints in mathematical programming

Feruccio Bilich, Department of Economics, Management and Industrial Engineering, University of Aveiro, Campus de Santiago, 3810-193, Aveiro, Portugal, bilich.feruccio@gmail.com

The weighting coefficients method consists in encoding directly into the objective function information which was usually encoded as constraints. The advantages of weighting coefficients in contraposition to the disadvantages of constraints are explained. It is shown how some special cases of objective functions such as multinomial distribution (discrete functions) and gamma and exponential distributions (continuous functions) with constraints can be represented by an equivalent function with weighting coefficients. Finally a general proof of the weighting coefficients method is derived.

2 - A global optimization approach for the equilibrium composition of natural silicate melts in magmatic fluids.

Andrea Cassioli, Dipartimento di sistemi e informatica, Universita' di Firenze, Via di S.Marta 3, 50139, Firenze, Italy, cassioli@dsi.unifi.it, *Marco Locatelli*, *Antonella Longo*, *Luca Consolini*

This work develops an exact method to find the global optimum of the Gibbs free energy for the equilibrium composition of natural silicate melts, whose efficient computation is required for any realistic numerical simulation of magma fluid-dynamics (the complexity of these natural phenomena requires advanced physical-mathematical models avoiding oversimplifications). We define an optimality-based domain reduction scheme, based on convex and linear models. The existence of a solution has been also
proved. Computational results will be shown to confirm the approach effectiveness.

3 - Large-scale decomposition with Fejer processes for convex programming

Andrey Velichko, Institute of Automation and Control Processes, Radio, 5, 690041, Vladivostok, Russian Federation, vandre@dvo.ru

Fejer processes with projection operator studied by Eremin and small decreasing disturbances approach analysed by Nurminski are used to design iterative and parallel algorithms for convex programming with large quantity of linear inequality constraints. Convex hull of projections onto disjoint subsets of a feasible region is used as Fejer operator. Selection of initial point, step adjustments and feasible set decomposition strategies are proposed to get linear convergence and nearly polynomial (in constraints) algorithm complexity of degree 4-5. MPI Toolbox package for Octave software is used.

4 - A Memetic Clonal Selection Algorithm for a Combinatorial Optimization Problem

Mario Pavone, Department of Mathematics and Computer Science, University of Catania, v.le A. Doria 6, 95125, Catania, Italy, mpavone@dmi.unict.it, *Natalio Krasnogor*

Fuel Distribution Problem(FDP) is a combinatorial optimization task closely related but not identical to the Capacitated Vehicle Routing Problem(CVRP), and can also be seen as a variant of the classical Multiple Container Packing Problem. The differences between FDP and CVRP are simple but crucial, because affect both on the optimization strategy and on the design of the instances. A memetic clonal selection algorithm is presented based on DFS algorithm and local search operator. The results show that presented algorithm seems to be efficient, in order to quality and homogeneity of solutions.

■ SE-05

Saturday, 15:00 - 17:00
23.3.9

Convex Analysis and Applications 4

Contributed session

Chair: *Miguel Goberna*, Estadística e Investigación Operativa, Universidad de Alicante, Ctra. San Vicente s/n, 03080, San Vicente del Raspeig, Alicante, Spain, mgoberna@ua.es

1 - Voronoi cells of arbitrary sets via linear inequality systems

Margarita Rodríguez Álvarez, Dpto. Estadística e Investigación Operativa, Universidad de Alicante, Ctra. Alicante-San Vicente s/n, 03690, San Vicente del Raspeig, Alicante, Spain, marga.rodriguez@ua.es, *Miguel Goberna*, *Virginia Vera De Serio*

Given a set T of the Euclidean space and a point s in T, the Voronoi cell of s is the set of points whose distance to s is not greater than its distance to any other point of T. The Voronoi diagram of T is the family of Voronoi cells of all the elements of T. There exists a wide literature on Voronoi diagrams of finite sets. In this talk we analyze the properties of the Voronoi cells when T is an infinite set taking into account that they can be expressed as solution sets of linear inequality systems indexed by T.

2 - New characterizations and conjugation scheme for e-convex functions

José Vicente-Pérez, Dept. Statistics and Operations Research, University of Alicante, 03690, San Vicente del Raspeig, Spain, Jose.Vicente@ua.es

A subset of a locally convex space is called evenly convex (or, in brief, e-convex) if it is the intersection of a family of open halfspaces. In the same way, an extended real valued function on such a space is called e-convex if its epigraph is e-convex. In this talk, we show that any e-convex function is Phi-convex, that is, it is the pointwise supremum of some family of elementary functions, and we define a new conjugation scheme. Also, we introduce a suitable support function for e-convex sets.

3 - Fenchel duality for evenly convex optimization problems

Maria Dolores Fajardo, STATISTICS AND OPERATIONAL RESEARCH, UNIVERSITY OF ALICANTE, Faculty of Sciences, Carretera San Vicente del Raspeig s/n - 03690 San Vicente del Raspeig, ALICANTE, ALICANTE, Spain,

md.fajardo@ua.es, *José Vicente-Pérez*, *Margarita Rodríguez Álvarez*

In this talk we deal with strong Fenchel duality for optimization problems where both inequality system and objective function are evenly convex. The number of constraints and the dimension of the topological vector space where all the involved functions are defined on are arbitrary, possibly infinite. To this aim, via perturbation approach, a conjugation scheme for evenly convex functions, based on generalized convex conjugation, is used. The key is to extend some well-known results from convex analysis, involving lower semicontinuous convex functions, to this more general framework.

4 - Optimality without constraint qualifications

Miguel Goberna, Estadística e Investigación Operativa, Universidad de Alicante, Ctra. San Vicente s/n, 03080, San Vicente del Raspeig, Alicante, Spain, mgoberna@ua.es

This work is based on the dual characterization, without constraint qualification nor closedness condition, of inequalities involving two convex functions defined on locally convex vector spaces. These results provide an asymptotic formula for subdifferentials of such special type of functions. The rest of the talk is devoted to applications of the previous results to several classes of optimization models, giving optimality and duality theorems on DC problems with convex constraints (including semidefinite ones), convex and semidefinite problems, and infinite linear problems.

■ SE-06

Saturday, 15:00 - 17:00

23.3.10

Interior Point Methods

Contributed session

Chair: *M. Fernanda P. Costa*, Department of Mathematics and Applications, University of Minho, Campus de Azur, 4800, Guimaraes, Portugal, mfc@mct.uminho.pt

1 - Analysis of the accuracy of the Interior-point methods based on the so-called kernel functions

Manuel V. C. Vieira, Mathematics, Universidade Nova de Lisboa, Faculdade de Ciências e Tecnologia, Quinta da Torre, 2829-516, Caparica, Portugal, mvcv@fct.unl.pt

Interior-point methods that use barrier functions induced by some real univariate kernel functions have been studied for the last decade. In these IPM's the algorithm stops when finds a solution such that is close (in the barrier function sense) to a point in the central path such that its duality gap is small enough. But this does not directly imply that we got a solution with the desired accuracy. Until now, this was not properly addressed. So, in this presentation we analyze the accuracy of the solution produced by the mentioned algorithm.

2 - An interior-point method for solving SDO problems from infeasible starting points

Hossein Mansouri, Applied mathematics, Shahrekord university, Dep. Applied mathematics, Shahrekord university, Shahrekord, Iran, Islamic Republic Of, h.mansouri@tudelft.nl, *Maryam Zangiabadi*

The algorithm uses two types of Nesterov-Todd steps which called feasibility step and centering steps. The starting point depends on a positive number ζ. The algorithm terminates in $O(nL)$ steps either by finding a solution or by detecting that the primal dual problem pair has no optimal solution $(X * \text{y} * \text{Ş}*)$ with vanishing duality gap such that $X * + S* \preceq \zeta I$.

3 - An interior-point algorithm for $P\_*(k)$-LCP with full-Newton steps

Maryam Zangiabadi, Applied Mathematics, Shahrekord University, Dep. Applied Mathematics, Shahrekord university, SHahrekord, Iran, Islamic Republic Of, m.zangiabadi@tudelft.nl, *Hossein Mansouri*

In this paper we propose an interior-point algorithm for $p\_*(k)$ linear complementarity problems ($p\_*(k)$-LCP). Since the algorithm uses only full-Newton steps, it has the advantage that no line searches are needed. Moreover, it is proven that the number of iterations of the algorithm is $O\left(\sqrt{n}\log\frac{n}{\varepsilon}\right)$, which coincides with the well-known best iteration bound for $p\_*(k)$-LCP.

4 - Performance of nonmonotone filter line search techniques in interior point barrier algorithms for nonlinear programming

M. Fernanda P. Costa, Department of Mathematics and Applications, University of Minho, Campus de Azur, 4800, Guimaraes, Portugal, mfc@mct.uminho.pt, *Edite M.G.P. Fernandes*

We analyze the performance of interior point nonmonotone filter line search algorithms for solving nonlinear optimization problems. However, filter method also prevents the rapid convergence of the methods by rejecting steps that make good progress. To address this issue, second-order corrections have been used to improve a direction when a trial point is rejected. A less demanding alternative considers nonmonotone strategies, which allow the filter measures to increase on certain iterations. Numerical results from two nonmonotone strategies are shown.

Saturday, 17:30 - 19:00

■ SF-03

Saturday, 17:30 - 19:00
23.3.4

Optimal Control 2

Contributed session

Chair: *Moulay Rchid Sidi Ammi*, Department of Mathematics, Faculté des Sciences et Techniques Errachidia, Morocco, B.P 509 Boutalamine, 52000, Errachidia, Morocco, sidiammi@ua.pt

1 - Asymptotic stability in optimal control problems with time delay

Musa Mammadov, Graduate School of Information Technology and Mathematical Sciences, University of Ballarat, University Drive, Mount Helen, P.O. Box 663, 3353, Ballarat, Victoria, Australia, m.mammadov@ballarat.edu.au

The problem of qualitative analysis of optimal trajectories for a special class of optimal control problems described by differential delay equations is considered. This kind of equations has attracted a significant interest in recent years due to their frequent appearance in a wide range of applications. They serve as mathematical models describing various real life phenomena in mathematical biology, population dynamics and physiology, electrical circuits and laser optics, economics, life sciences and others.

2 - Optimal control model for crude oil operations in refineries and ports

Joao Lauro D. Faco', Dept. of Computer Science, Universidade Federal do Rio de Janeiro, Av. do PEPE, 1100 / 203, 22620-171, Rio de Janeiro, RJ, Brazil, jldfaco@ufrj.br, *Fabio Fagundez*, *Adilson Elias Xavier*

Crude oil scheduling is modeled as a nonlinear optimal control problem, covering crude oil transfer operations from tankers to crude distillation units. The system infrastructure is modeled as a graph with equipments as nodes and connections as arcs. Each connection can have a transfer operation flow which is mapped to a control variable, whereas equipment contents are mapped to state variables. Yes-No decisions are modeled as complementarity constraints, thus achieving a continuous model. The approach is tested on examples from the literature.

3 - Optimal Control of Non-Local Thermistor Equations

Moulay Rchid Sidi Ammi, Department of Mathematics, Faculté des Sciences et Techniques Errachidia, Morocco, B.P 509 Boutalamine, 52000, Errachidia, Morocco, sidiammi@ua.pt, *Delfim F. M. Torres*

In this talk we are concerned with the optimal control problem of the well known non-local thermistor problem. Existence and regularity of the optimal control are established. Moreover, the energy estimates and the obtained class of weak solutions allow us to derive the optimality system for our optimal control problem. The obtained optimality conditions consist of a system of parabolic equations. An other part of our work is devoted to the uniqueness of solutions of the optimality system and therefore the uniqueness of the optimal control. Finally, some numerical results are shown.

■ SF-06

Saturday, 17:30 - 19:00
23.3.10

Nonsmooth Optimization 2

Contributed session

Chair: *Vicente Novo*, Matematica Aplicada, Universidad Nacional de Educacion a Distancia, Juan del Rosal no. 12, 28040, Madrid, Spain, vnovo@ind.uned.es

1 - Sample Average Approximation for Mean Stochastic Quasigradients

James Blevins, STATISTICS DEPARTMENT (Ekonomikum), Uppsala University, Box 513, 751 20, UPPSALA, Sweden, James.Blevins@statistics.uu.se

We study the expected error of the sample average approximation (SAA), following Nemirovskii, Shapiro, etc. We relax prior hypotheses, e.g. independent sampling, uniform convexity, & Gateaux differentiability. Applications include stochastic programming & statistics (estimating functions & M-estimation). For minimizing sums of functions, the SAA method has advantages over 'incremental' methods. For the sum of subdifferentials & epigraphs in "disaggregated' bundle methods, we present a sparse convex representation (synthesizing lemmas of Carathéodory, Rådström, and Shapley-Folkman).

2 - Solving Different Clustering Problem Formulations by Hyperbolic Smoothing and Partition into Boundary and Gravitational Regions

Adilson Elias Xavier, Graduate School of Systems Engineering and Computer Scinces, Federal University of Rio de Janeiro, P.O. Box 68511, Ilha do Fundão - Centro Tecnologia - H319, 21941-972, Rio de Janeiro, RJ, Brazil, adilson@cos.ufrj.br, *Vinicius Layter Xavier*

The Hyperbolic Smoothing Clustering Method adopts a smoothing strategy that solves a sequence of differentiable unconstrained optimization problems. The paper presents a new idea: the partition of the set of observations into two non overlapping parts. The first set, named boundary band zone, corresponds to the observation points relatively close to two or more centroids. The second set, named gravitational points, corresponds to observation points significantly closer to a single centroid. The combination of the two methodologies drastically simplify the computational tasks.

3 - Free boundary problem for the discretised heat equation

Imme van den Berg, University of Évora, 7000, Évora, ivdb@uevora.pt

We consider a discretization for the free boundary problem for the backward heat equation with finite horizon. Under appropriate conditions the domain of the solution is divided by a free boundary into a continuation region where the solution of the heat equation is maximal and a stopping region where a prescribed function is maximal. The boundary is free in the sense that it is a priori unknown and must be determined simultaneously with the solution. Inspired by results of Gevrey, Friedman and Van Moerbeke, we study regularity properties of the solution and the free boundary.

Saturday, 19:00 - 19:15

■ SG-01

Saturday, 19:00 - 19:15
23.1.5

Closing Session

Plenary session

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